# Supporting Information for "When will humanity notice its influence on atmospheric rivers?"

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**Introduction** In this supplementary information, we include the detailed derivations for estimating the confidence intervals of the forced response in a large ensemble dataset (Text S1), figures to support the finding in the main text, such as the analysis based on 85% IVT threshold (Figs. S2-S3), different regression approaches for approximating $f(n_t)$ in Eq. (4) (Figs. S4-S5), and the results based on two other AR detection algorithms (Figs. S6-S7). We also inlcude results from all natural forcing simulations (Fig. S8) and show that the observed trend is caused by the radiative forcing of anthropogenic greenhouse gases rather than the climatological drift in SPEAR.

## Text S1. Estimating the Confidence Interval of Forced Response in Large Ensemble Data

In this supporting material, we provide two different ways of quantifying the confidence interval of the forced response in a large initial-condition ensemble. We will demonstrate that the two approaches lead to the same conclusion when the sample size (data length) is big enough (i.e., $\geq 30$). We first start with a linear regression model. In a linear regression model,

$$\boldsymbol{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\xi} \tag{1}$$

where $\boldsymbol{y}$ is the predictand, $\mathbf{X}$ is the predictor matrix, $\boldsymbol{\beta}$ is the vector of regression coefficients and $\boldsymbol{\xi}$ is the residual term. The best estimation of $\boldsymbol{\beta}$ (i.e., $\hat{\boldsymbol{\beta}}$) in a linear least squared sense can be acquired with the following equation

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\boldsymbol{y} \tag{2}$$

In the most ideal case, $\boldsymbol{\xi}$ is normally distributed and decorrelated with $\mathbf{X}$. Then we can find the uncertainty in $\hat{\boldsymbol{\beta}}$ caused by $\boldsymbol{\xi}$ is also normally distributed and centered around $\boldsymbol{\beta}$ (i.e., unbiasedness) with a variance of $(\mathbf{X}^T\mathbf{X})^{-1}\sigma^2$ (i.e., $\hat{\boldsymbol{\beta}} \sim \mathcal{N}(\boldsymbol{\beta}, \frac{\sigma^2}{\mathrm{Var}(\mathbf{X})})$), where $\sigma^2$ is the variance of $\boldsymbol{\xi}$. This can be proved by estimating the expected values of $\hat{\boldsymbol{\beta}}$ and $(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})^T(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$

$$\begin{aligned} \mathrm{E}[\hat{\boldsymbol{\beta}}] &= \mathrm{E}[(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\boldsymbol{y}] \\ &= \mathrm{E}[(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T(\mathbf{X}\boldsymbol{\beta} + \boldsymbol{\xi})] \\ &= \boldsymbol{\beta} + \mathrm{E}[(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\boldsymbol{\xi}] \\ &= \boldsymbol{\beta} \end{aligned} \tag{3}$$

and

$$\begin{aligned} \mathrm{E}[(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})^T(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})] &= \mathrm{E}[(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\boldsymbol{\xi}((\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\boldsymbol{\xi})^T] \\ &= \mathrm{E}[(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\boldsymbol{\xi}\boldsymbol{\xi}^T\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}] \\ &= \sigma^2\mathrm{E}[(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}] \\ &= (\mathbf{X}^T\mathbf{X})^{-1}\sigma^2 \end{aligned} \tag{4}$$

In S3 and S4, $\mathrm{E}[A]$ indicates the expected value of $A$. By replacing $\mathbf{X}$ in S4 with $\mathbf{n_t}$ (i.e., time steps), we can find equation S4 is similar to equation 2 of Thompson et al. (2015) except in a variance form. Equations S1-S4 suggest that given the linear assumption of the forced response, the confidence intervals of $\boldsymbol{\beta}$ can be written as:

$$\mathrm{CI}_\beta = \boldsymbol{\beta} \pm t\frac{\sigma}{\sqrt{\mathbf{X}^T\mathbf{X}}} \tag{5}$$

where $t$ is the t-statistics for the desired significance levels. Equation S5 is equivalent to equation 1 in Thompson et al. (2015).

## Method I: Projecting model residual onto the forced response

Since the above equations are based on linear assumption, this section will explore a more general form of equation S1 and find the corresponding confidence intervals of the forced response. We adopt equation 2 in the main text and consider the case where the deviation of AR-day seasonal frequency from a reference year ($\boldsymbol{y}$) is determined by two components:

$$\boldsymbol{y} = f(\mathbf{n_t}) + \boldsymbol{\xi} \tag{6}$$

where $f(\mathbf{n_t})$ is the forced response as a function of $\mathbf{n_t}$. Different from equation S1, no linear assumption is made for $f(\mathbf{n_t})$. Again, in the most ideal case, $\boldsymbol{\xi}$ in equation S6 is decorrelated with $f(\mathbf{n_t})$. However, in some cases, such as a short data length or small ensemble sizes, $\boldsymbol{\xi}$ and $f(\mathbf{n_t})$ can be partially correlated and lead to the uncertainty of estimating $f(\mathbf{n_t})$. Let $\boldsymbol{\xi}$ to be the combination of two different components

$$\boldsymbol{\xi} = \alpha f(\mathbf{n_t}) + \boldsymbol{\eta} \tag{7}$$

where $\alpha f(\mathbf{n_t})$ is the part correlated with $f(\mathbf{n_t})$ and $\boldsymbol{\eta}$ is the uncorrelated part. Physically, the first term of equation S7 represents the projection of noise from internal climate variability onto the forced response. By rewriting equation S6 with S7, we have equation S8

$$\boldsymbol{y} = (1 + \alpha) f(\mathbf{n_t}) + \boldsymbol{\eta} \tag{8}$$

From equation S8, one can find that the confidence interval of $f(\mathbf{n_t})$ is determined by $\alpha$. Thus, we will next estimate the mean and variance of $\alpha$ as done in equations S3 and S4

$$
\begin{aligned}
\mathrm{E}[\hat{\alpha}] &= \mathrm{E}[(f(\mathbf{n_t})^T f(\mathbf{n_t}))^{-1} f(\mathbf{n_t})^T \boldsymbol{\xi}] \\
&= \mathrm{E}[(f(\mathbf{n_t})^T f(\mathbf{n_t}))^{-1} f(\mathbf{n_t})^T (\alpha f(\mathbf{n_t}) + \boldsymbol{\eta})] \\
&= \alpha + \mathrm{E}[(f(\mathbf{n_t})^T f(\mathbf{n_t}))^{-1} f(\mathbf{n_t})^T \boldsymbol{\eta}] \\
&= \alpha
\end{aligned} \tag{9}
$$

and

$$
\begin{aligned}
\mathrm{E}[\mathrm{Var}(\hat{\alpha})] &= \mathrm{E}[(f(\mathbf{n_t})^T f(\mathbf{n_t}))^{-1} f(\mathbf{n_t})^T \boldsymbol{\xi}((f(\mathbf{n_t})^T f(\mathbf{n_t}))^{-1} f(\mathbf{n_t})^T \boldsymbol{\xi})^T] \\
&= \mathrm{E}[(f(\mathbf{n_t})^T f(\mathbf{n_t}))^{-1} f(\mathbf{n_t})^T \boldsymbol{\xi}\boldsymbol{\xi}^T f(\mathbf{n_t})(f(\mathbf{n_t})^T f(\mathbf{n_t}))^{-1}] \\
&= \sigma^2 \mathrm{E}[(f(\mathbf{n_t})^T f(\mathbf{n_t}))^{-1} f(\mathbf{n_t})^T f(\mathbf{n_t})(f(\mathbf{n_t})^T f(\mathbf{n_t}))^{-1}] \\
&= (f(\mathbf{n_t})^T f(\mathbf{n_t}))^{-1} \sigma^2
\end{aligned} \tag{10}
$$

Equation S9 and S10 suggest that $\hat{\alpha} \sim \mathcal{N}(\alpha, \frac{\sigma^2}{\mathrm{Var}(f(\mathbf{n_t}))})$. In the case where $f(\mathbf{n_t})$ is decorrelated with $\boldsymbol{\xi}$, $\mathrm{E}[\hat{\alpha}]$ is 0. This also indicates that $\hat{\alpha} \sim \mathcal{N}(0, \frac{\sigma^2}{\mathrm{Var}(f(\mathbf{n_t}))})$. The confidence intervals of $f(\mathbf{n_t})$ can be written as

$$
\mathrm{CI}_f = f(\mathbf{n_t}) \pm t \frac{\sigma}{\sqrt{\mathrm{f}(\mathbf{n_t})^{\mathrm{T}} \mathrm{f}(\mathbf{n_t})}} f(\mathbf{n_t}) \tag{11}
$$

where $t$ is the t-statistics for the desired significance levels. Equation S11 is a more general form of S5 without hypothesising a linear forced response. One can prove that equation S5 and S11 are mathematically identical by substituting $f(\mathbf{n_t})$ with $\mathbf{n_t}\boldsymbol{\beta}$.

**Method II: CIs based on Central Limit Theorem (CLT)** Another option for estimating CIs of the forced response is by using Central Limit Theorem (CLT). CLT describes that when independent random variables are added (generating a synthesized distribution), their properly normalized sum tends toward a normal distribution even if the original variables themselves are not normally distributed. The mean of the synthesized distribution ($\bar{X_n}$) is equal to the mean of independent variables ($\mu$) and the standard deviation will be reduced by a factor of $\sqrt{n}$ (i.e, $\sigma_{\bar{X_n}} = \frac{\sigma}{\sqrt{n}}$, where $\sigma_{\bar{X_n}}$ is the standard deviation of the synthesized distribution, $\sigma$ is the standard deviation of independent variables and $n$ is the sample size).

In our study, the ensemble mean is considered as the forced response. The corresponding CIs based on CLT can therefore be written as

$$\mathrm{CI}_{f(\mathbf{n_t})} = f(\mathbf{n_t}) \pm t\frac{\sigma}{\sqrt{n}} \tag{12}$$

where $f(\mathbf{n_t})$ is the forced response, $t$ is the t-statistics for desired significance level, $\sigma$ is the standard deviation of detrended ensemble spread and $n$ is the ensemble size. Comparing to equations S5 and S11, we can find the major difference is in the denominator of the second term. For method I, the uncertainty is reduced when longer data is used, while it is determined by ensemble size in method II. For estimating time of signal emergence, method I is physically more consistent with our purpose than method II. The reason is that we wish to evaluate the signal in relation to the expected noise in a single realization of nature, and so we do not wish to evaluate the signal to noise that has been damped by the averaging across ensemble members. Equation S5 suggests that the unforced component (i.e., $\boldsymbol{\xi}$ in S1) leads to bigger uncertainty if the period of interest is shorter. However, this lead time-dependent factor is not included in method II. Figure S1 demonstrates the CIs given by methods I and II. One can clearly observe that the CIs based on method I are wider (more uncertain) at short lead times while the CIs based on method II are quite stationary over the whole period. With the increase of lead time, the CIs given by two methods become closer and two distributions are indistinguishable from each other.

**References**

Thompson, D. W. J., Barnes, E. A., Deser, C., Foust, W. E., & Phillips, A. S. (2015). Quantifying the role of internal climate variability in future climate trends. *Journal of Climate*, *28*(16), 6443 - 6456. Retrieved from `https://journals.ametsoc.org/view/journals/clim/28/16/jcli-d-14-00830.1.xml` doi: 10.1175/JCLI-D-14-00830.1

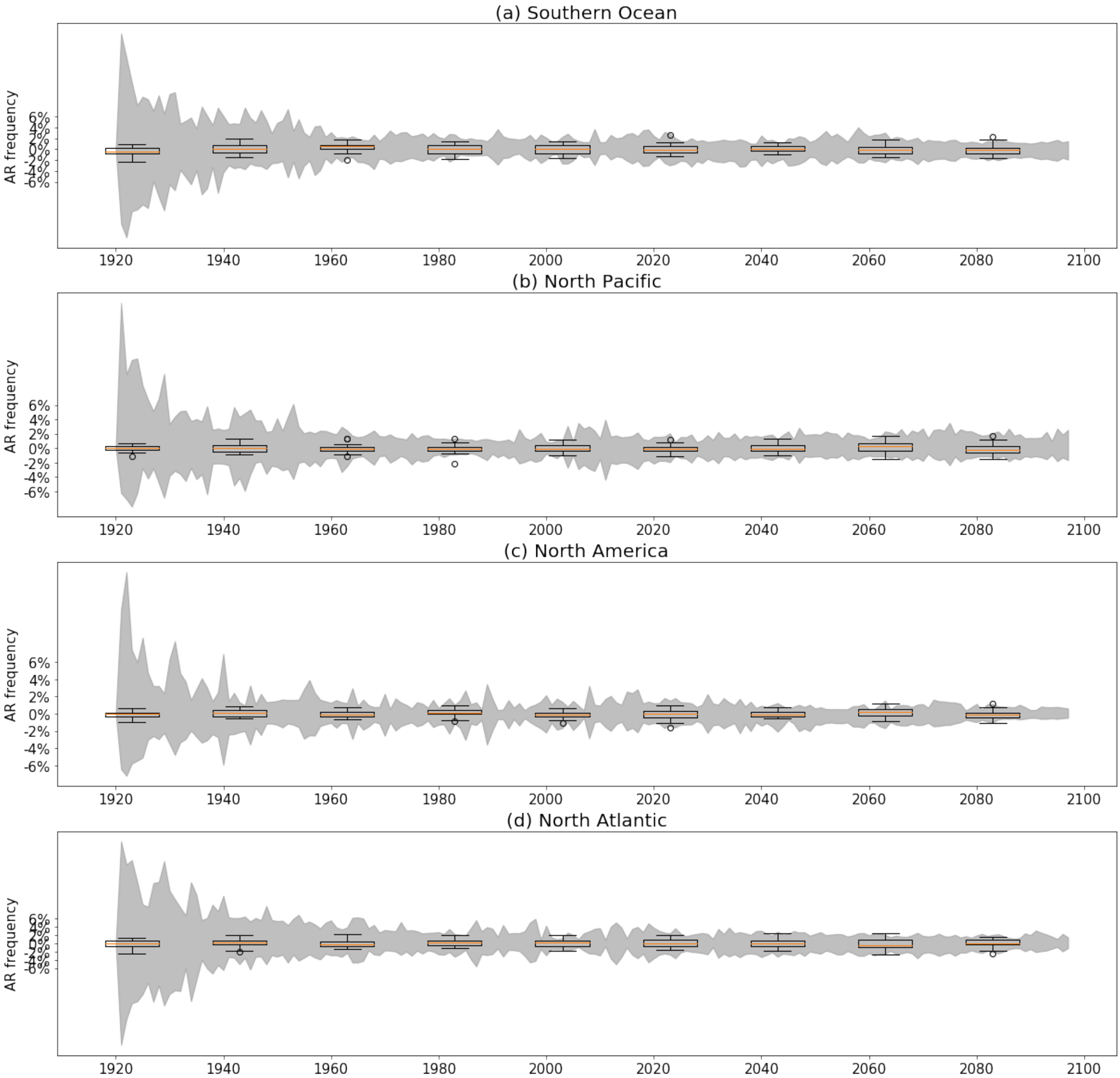


**Figure S1.** The CIs for $f(\mathbf{n_t})$ (with ensemble mean removed) based on projection method (method I, gray shading) and Central Limit Theorem (method II, box plots) over four locations: (a) Southern Ocean, (b) North Pacific, (c) North America, and (d) North Atlantic. The data contains both historical simulations (1921-2014) and SSP5-8.5 simulations (2015-2100).

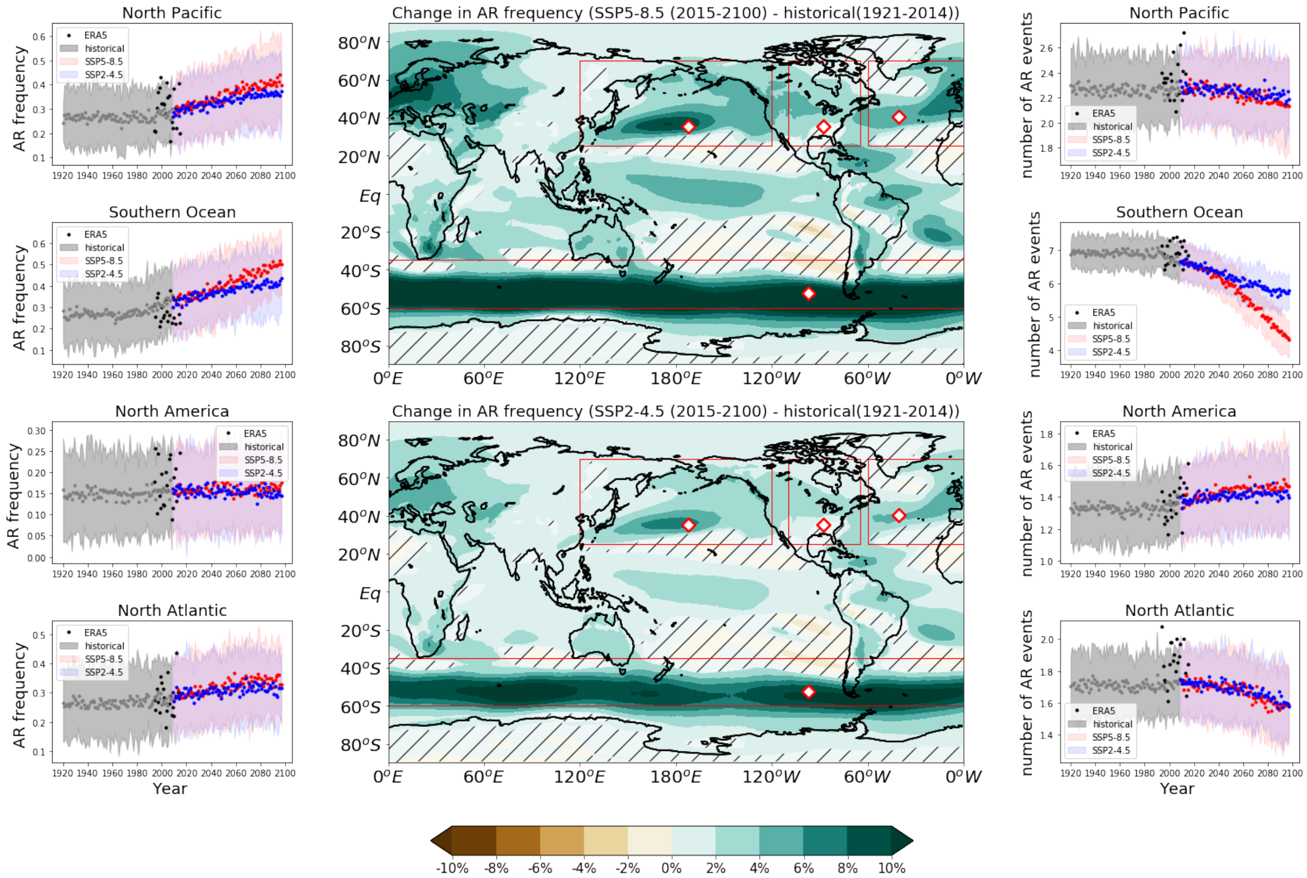


**Figure S2.** The same as Fig. 1 in the main text, except for 85% IVT threshold for AR detection.

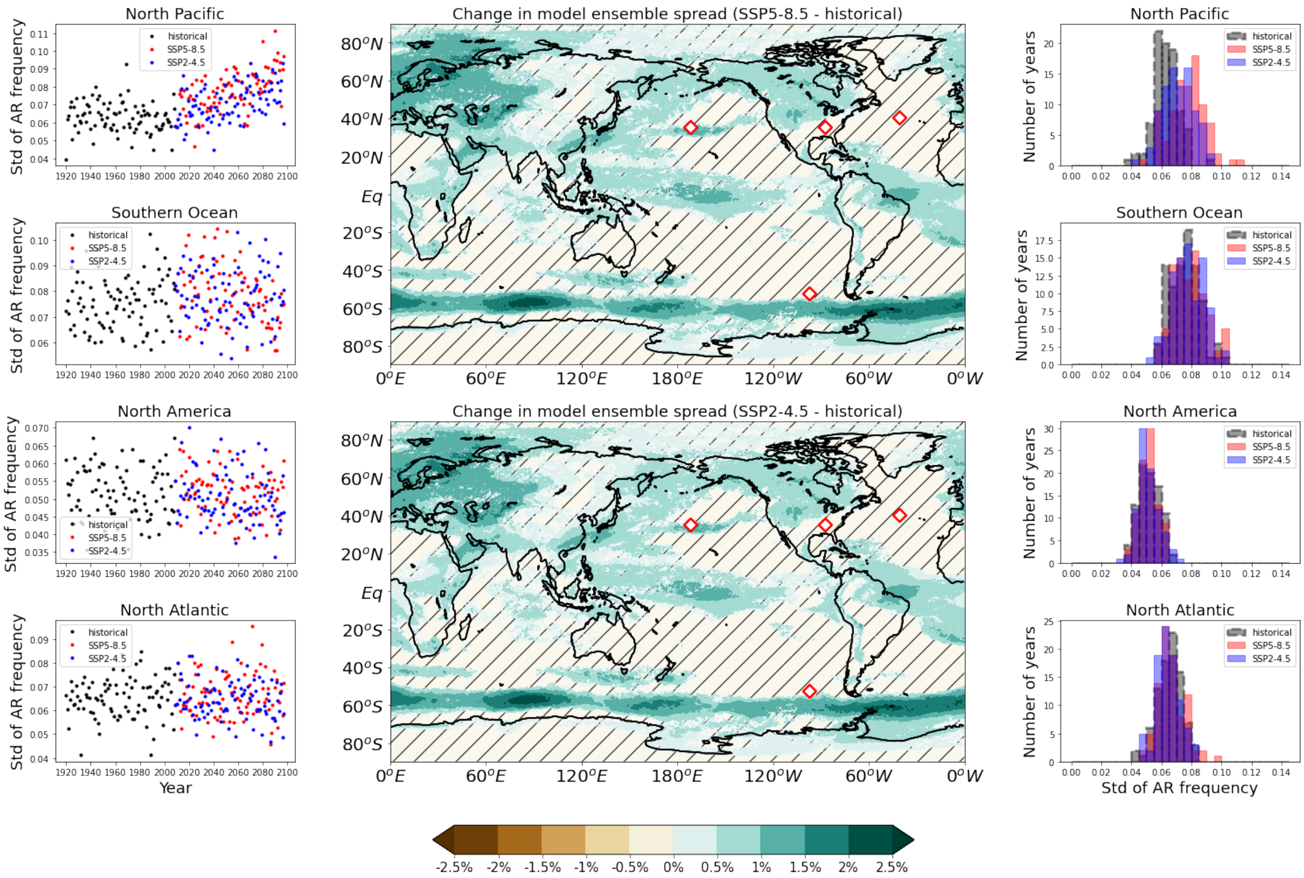


**Figure S3.** The same as Fig. 2 in the main text, except for 85% IVT threshold for AR detection.

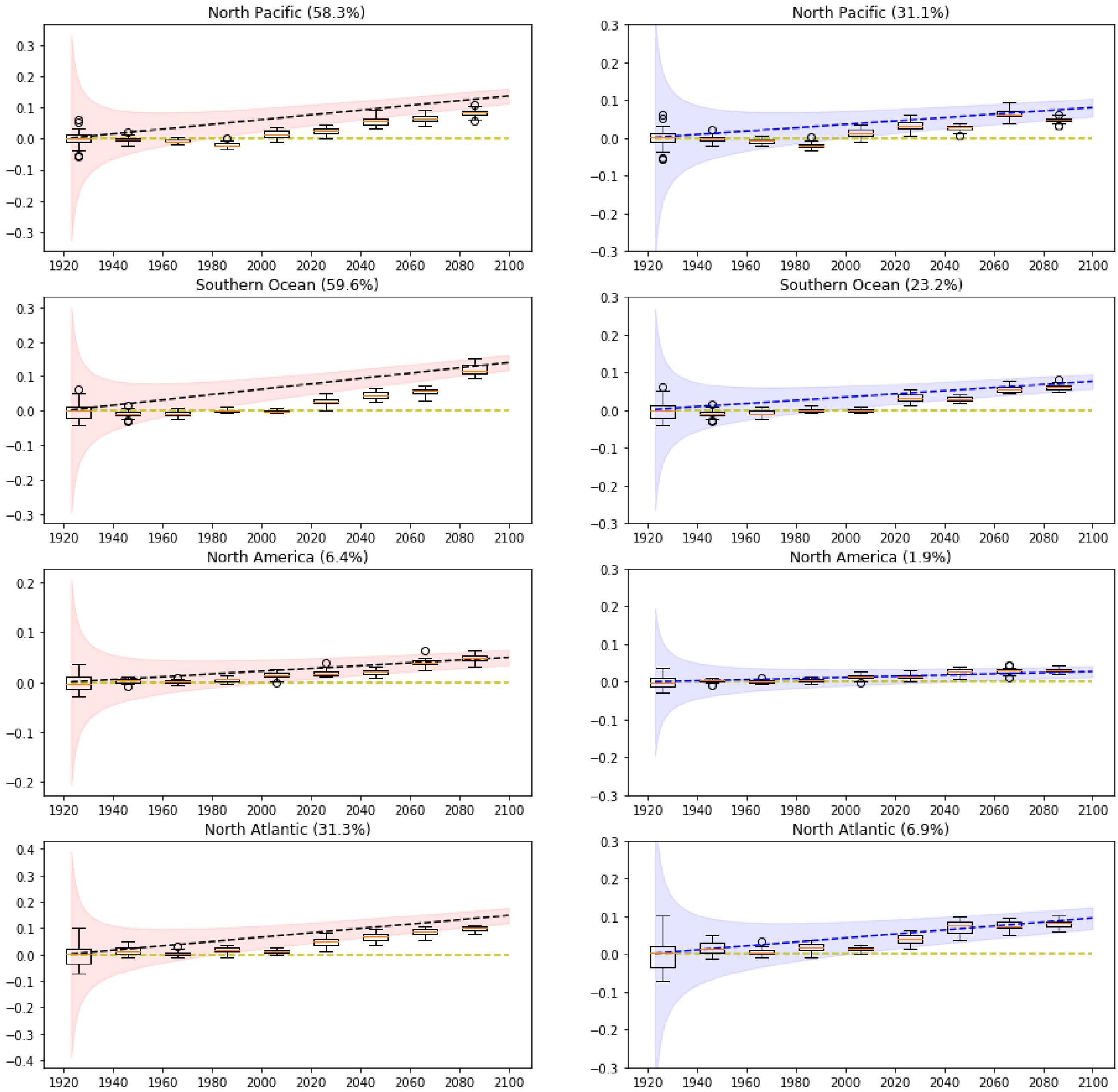


**Figure S4.** The same as Fig. 3 in the main text, except using linear regression for $f(n_t)$.

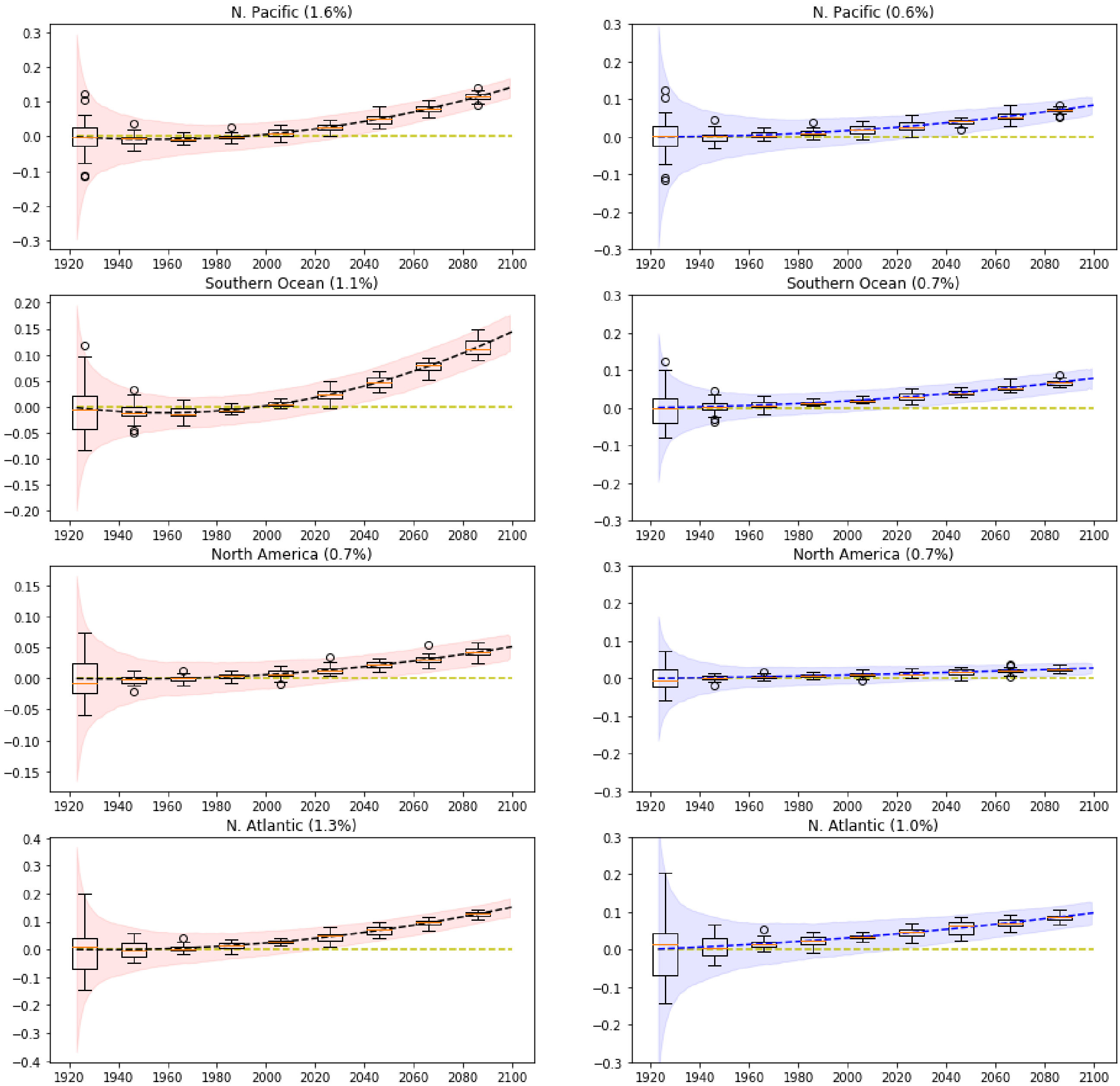


**Figure S5.** The same as Fig. 3 in the main text, except considering $s$ in Eq. (4) as a function of time.

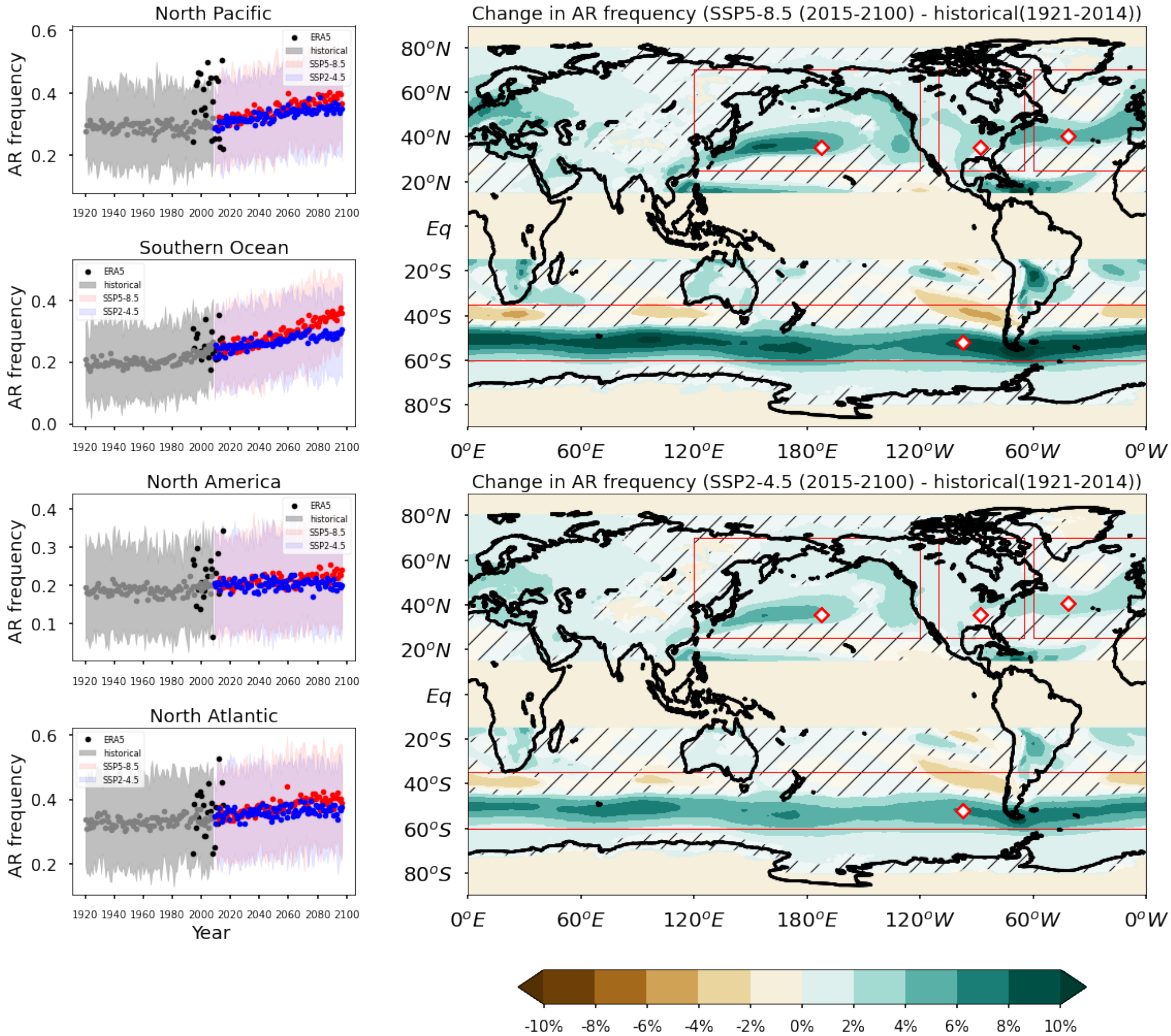


**Figure S6.** The same as Fig. 1 in the main text, except using the Tempest AR detection algorithm. The implemented criteria ensures $\sim$300 $\mathrm{kg \cdot m^{-1} s^{-1}}$ in IVT difference between the core of an AR and its environment.

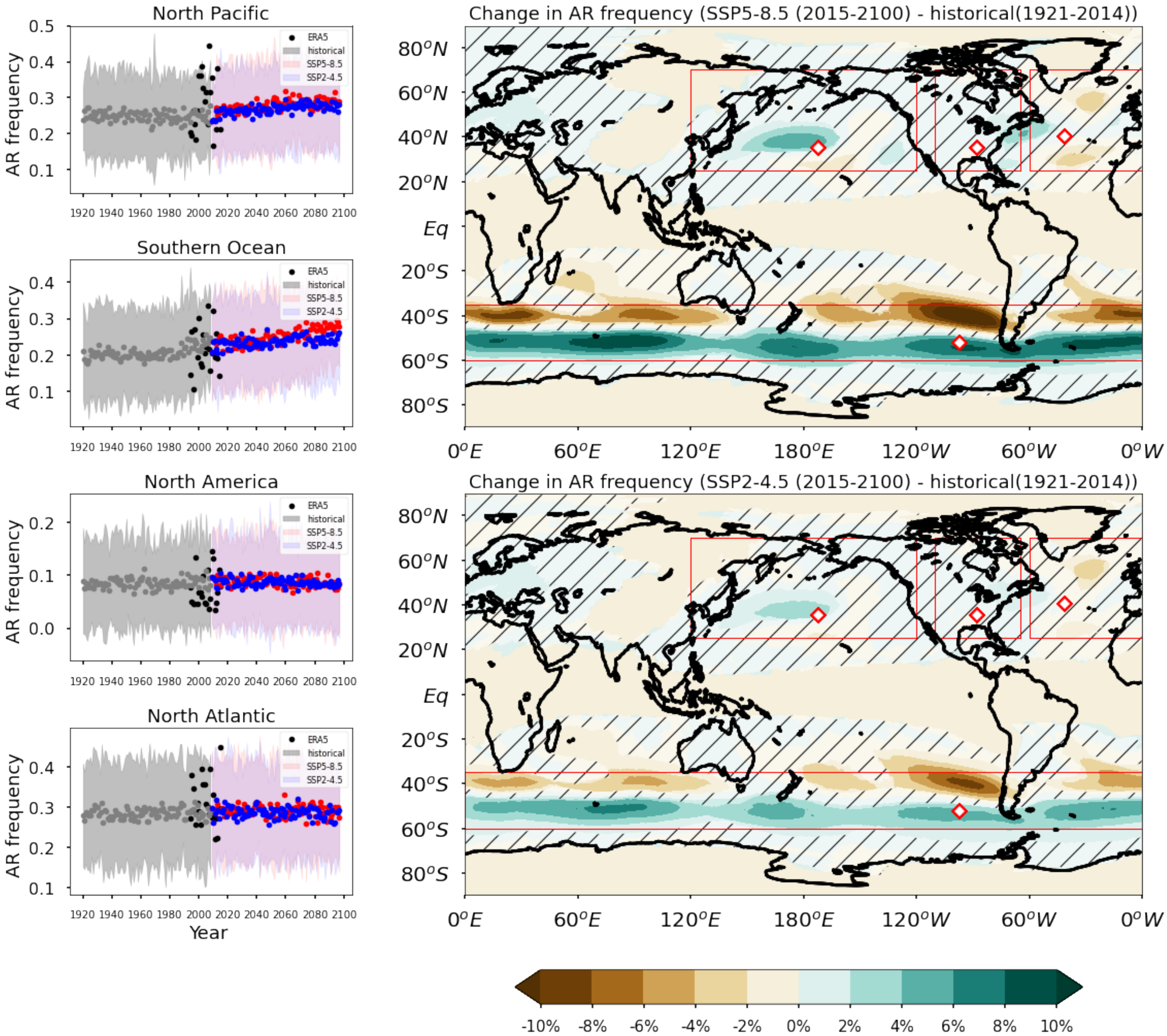


**Figure S7.** The same as Fig. 1 in the main text, except using the TECA-BARD AR detection algorithm with the constraint that ARs occupy a maximum planetary area of ∼5%.

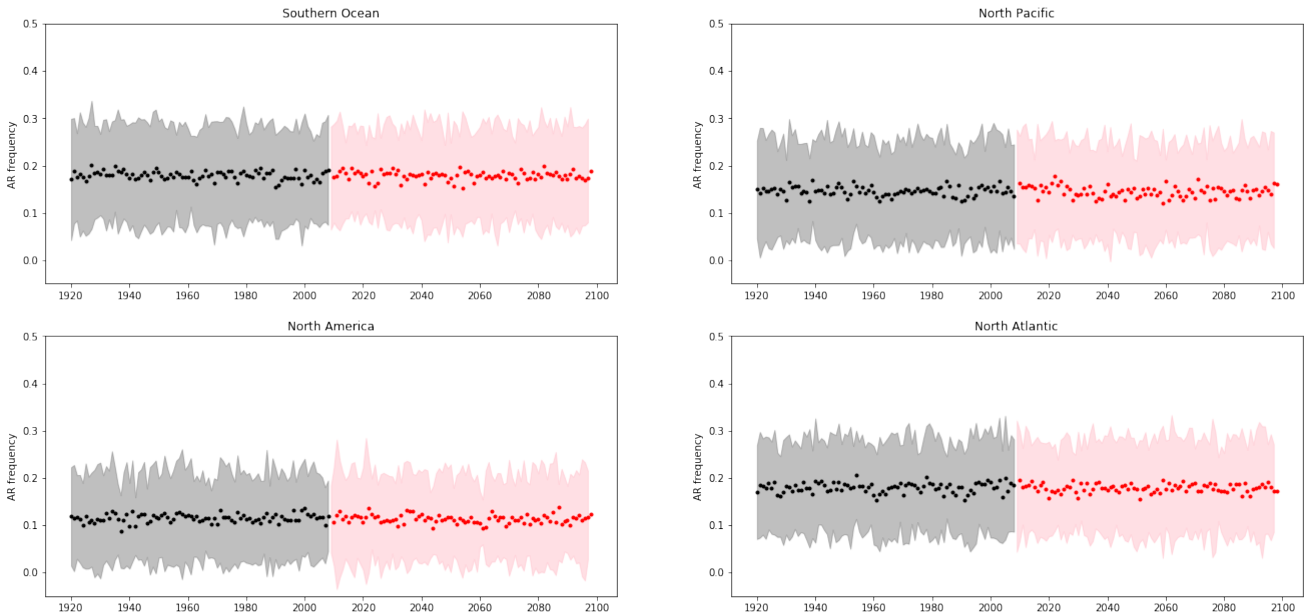


**Figure S8.** The AR frequency from all natural forcing simulations in four chosen locations (diamond marks in the middle column of Fig. 1) (a)Southern Ocean, (b)North Pacific (c)North America, and (d)North Atlantic. Scatters are ensemble mean and the shading shows 2 standard deviations of the ensemble spread.