ENSO's Decadal Dance viewed through a Local Lyapunov Lens

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1. Problem Statement

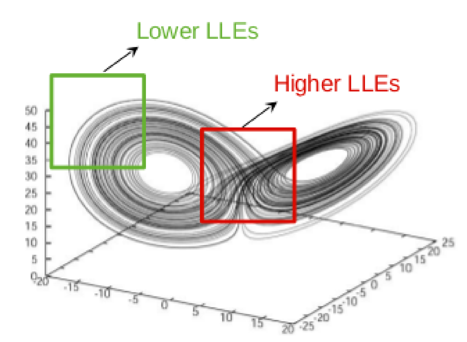
Decadal variability of ENSO is present in historical and paleo records, and has been simulated by a hierarchy of dynamical and statistical models. Predictability of ENSO varies with forecast lead-times, amplitude of interannual ENSO variability, and with decade. The limits of predictability depend on the mechanisms responsible for ENSO irregularity (chaos, noise), and equilibration at finite amplitude (stability of atmosphere-ocean interactions) (Sarachik & Cane, 2010).

In an early report, Abarbanel et al. (1991) note that **"the real issue of predictability is whether the atmosphere-ocean system constitutes a chaotic dynamical system at all time-scales"**, and point out that **"extremely long (1000 years or more) runs of coupled atmosphere-ocean models are required to study the issue of whether the [climate] system exhibits chaotic behavior at all time scales of interest (...)"**. Fortunately, such investigations are gradually becoming possible in long runs of coupled GCMs, such as the GFDL CM2.1 pre-industrial run (Wittenberg et al. 2006; Wittenberg 2009).

In this work, we investigate the limits of predictability in the context of dynamical systems theory. We use the Local Lyapunov Exponents (LLEs) of the NINO3 time series as a measure of predictability of the ENSO system in long control runs of GFDL CM2.1 and the ZC model. We explore the (multi)decadal variability of predictability and its links to the magnitude and frequency of events, as well as the possible culprits for this variability.

2. Ergodic theory of dynamical systems.

- The dynamics of a complex system can be captured by time-delay embedding of a single variable.
- Global Lyapunov Exponents measure the average rate of divergence of nearby trajectories in the phase-space, i.e. they characterize the average predictability of the attractor.



Prediction Error:

$$E(t) = E(0)exp(\lambda_1 t), \qquad (1)$$

λ_1: largest global exponent

Local Lyapunov Exponents $_a(\mathbf{x}, L)$ measure the growth or decay over L time steps of a perturbation made around a point $\mathbf{x}$ of the attractor:

As

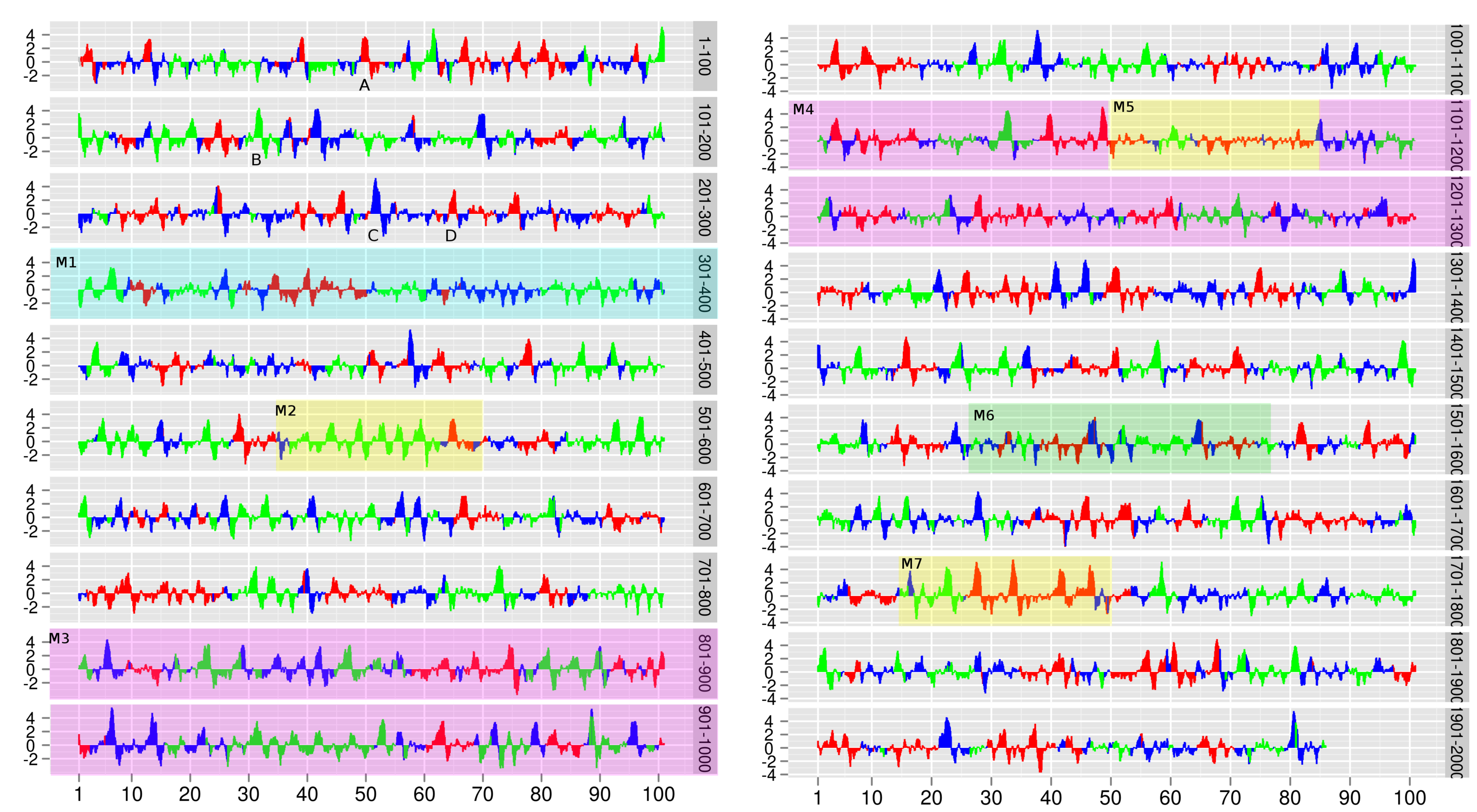
$$L \rightarrow \quad , \quad \lambda_a(\mathbf{x}, L) \longrightarrow \lambda_{global} \qquad (2)$$

Local error-doubling time:

$$(\lambda_1(\mathbf{x}, L))^{-1} \qquad (3)$$

LLEs are useful in characterizing predictability locally in the attractor of a weakly chaotic system that likely passes through phases of increased or decreased predictability (like ENSO).

3. ENSO Predictability in CM2.1-1860



Colors indicate levels of predictability, as determined by the LLEs.
Green → blue → red = more → less predictable (in 9% increments)

*Data and Methods Summary:

Monthly NINO3 Index from the GFDL CM2.1-1860 simulation. *NINO3 observations:* NOAA Extended Reconstructed SST (ERSST.v3) record. Phase-space reconstruction & Lyapunov Exponent calculation as per Brown et al. (1991), Abarbanel et al. (1992). LLEs are calculated on the NINO3 Index, with embedding dimension $m = 5$, time lag $\tau = 11$ months, scale of integration $L = 4$ months. The LLEs describe local error growth from small perturbations over a 4-month window. Comparison between model and observations is based on LLE calculations in 100-yr long samples. The investigation of '1997-98 events' in the model is based on LLE estimates in the first 500 years of heat content (0-300m, averaged over the Pacific 10S-10N), NINO3, air-sea fluxes, and wind.

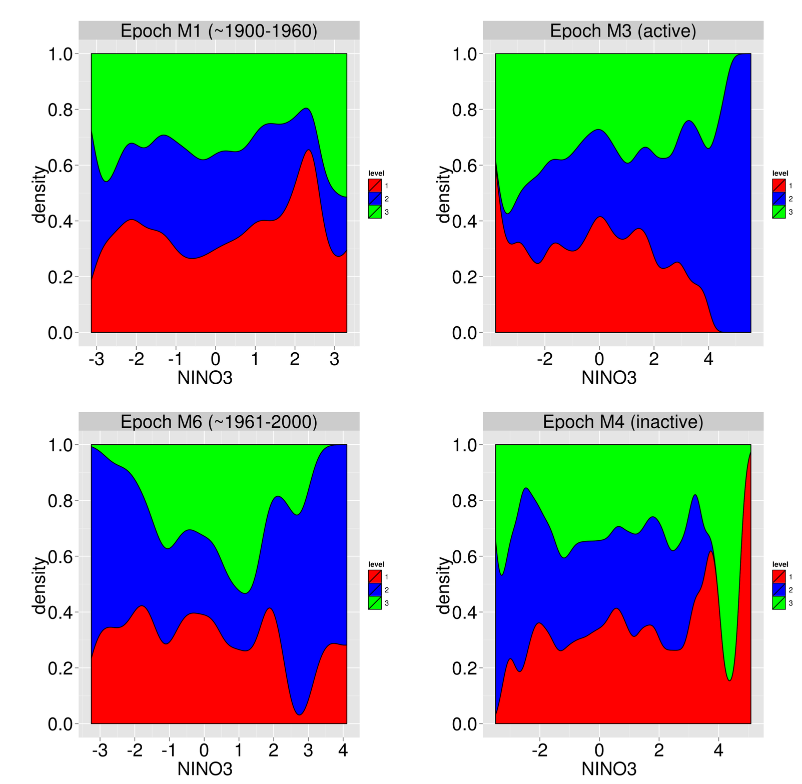
What do the LLEs of the monthly NINO3 Index mean?

The LLEs describe the capacity of tiny perturbations of the monthly NINO3 Index (e.g. WWBs) to grow at faster or slower rates due to the system's internal nonlinearities, signifying decreased or enhanced predictability.

4. Decadal variability in predictability

- Periods with sinusoidal moderate events (M2) are deemed the most predictable.
- Silent periods (M5) or irregular mega-ENSO periods (M7) are the least predictable.
- Irregularity in predictability, with decadal persistence, marks epochs with pre-1960 and post-1960 characteristics (M1 and M6, respectively).
- 'Active' periods (M3) are on average more predictable than 'inactive' ones (M4).

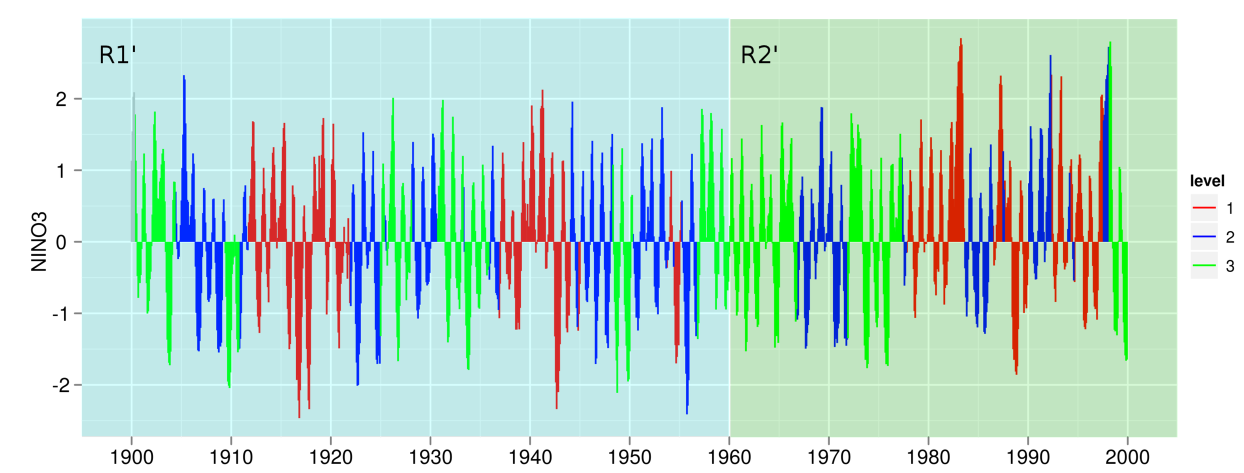
5. Predictability-magnitude relations vary by epoch.

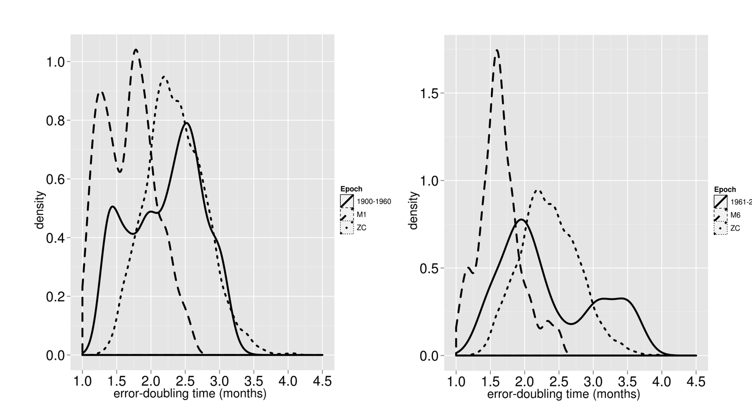


The figures show the probability that an event of certain magnitude is associated with a certain level of predictability.

- Strong El Niño events are more predictable in the epoch that 'mimics' the post-1960 observations (M6).
- El Niño predictability is enhanced during 'active' periods (M3) compared to 'inactive' ones (M4).

6. Model vs. Observations:
Who sets the upper bound of predictability?





Nature seems to be setting the upper bound of predictability for the GCM. The intermediate ZC model has error-doubling times closer to observed, but less variable.

- The median error-doubling time in the GCM (1.6 months) is less than the observed (2.2 months).
- Less variability in predictability in the model vs. observations.
- Predictability reduces from 1900-1960 to 1961-2000 by 8.5% (in the model 4%).
- But, heavier tails indicate cases of increased predictability in the record.

This result is not far from the reality of ENSO predictions:
Simple (even naive statistical) models often make more skillful predictions.

Acknowledgments

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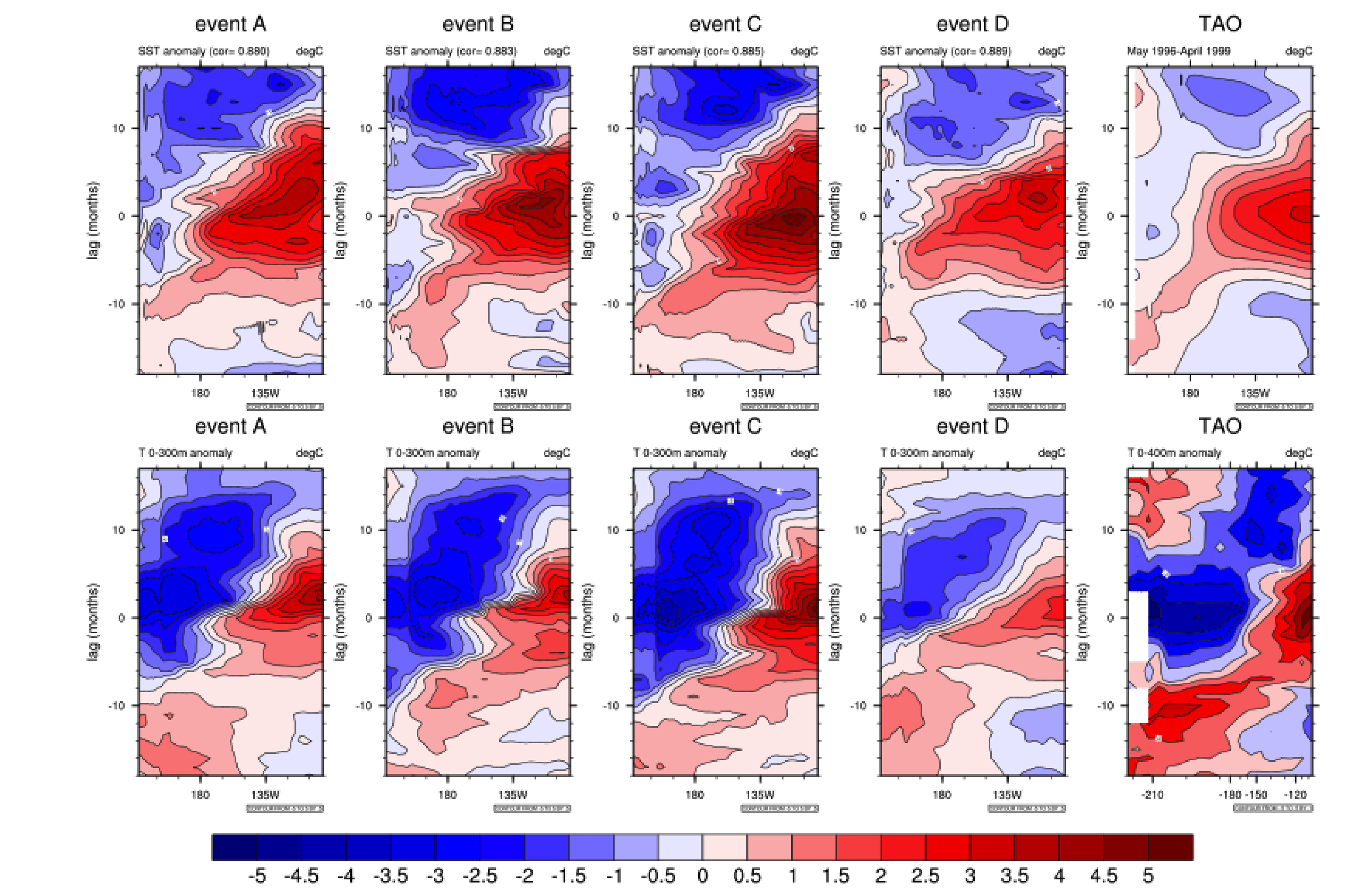
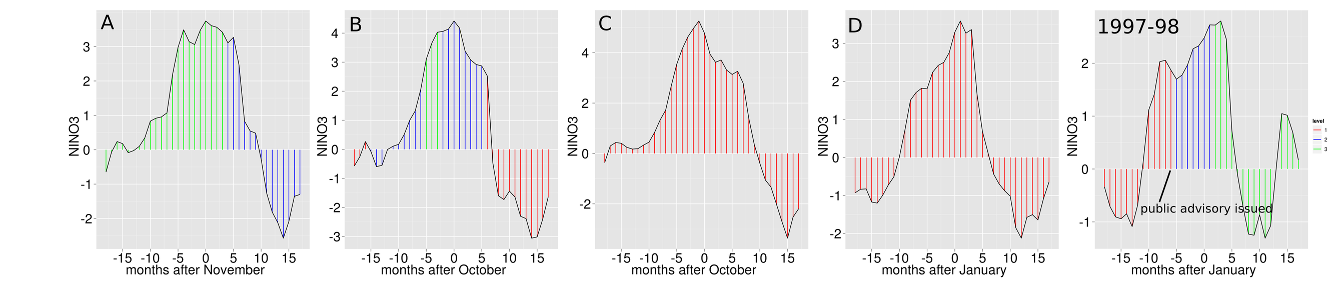
References: Abarbanel, H. et al (1991), *Issues in Predictability*, Report JASON Program Office; Abarbanel, H. et al. (1992), *Local Lyapunov Exponents from observed data*, J. Nonl. Sci., 2; Brown, R. et al. (1991), *Computing the Lyapunov Spectrum of a dynamical system from observed time series*, Phys. Rev A, 43; Sarachik,E.D. & M.A. Cane (2010), *The El Niño-Southern Oscillation Phenomenon*, Cambridge University Press; Wittenberg, A. T., A. Rosati, N.-C. Lau, and J. J. Ploshay (2006), *GFDL's CM2 global coupled climate models. Part III: Tropical Pacific climate and ENSO*, J. Climate, 19, 698-722; Wittenberg A.T. (2009), *Are historical records sufficient to constrain ENSO simulations?*, Geophys. Res. Lett. 36; Zebiak, S.E. & M.A. Cane (1985), *A Model El Niño Southern Oscillation*, Science, 228, p1085.

7. '1997-98 events' in CM2.1:
What dictates their predictability?





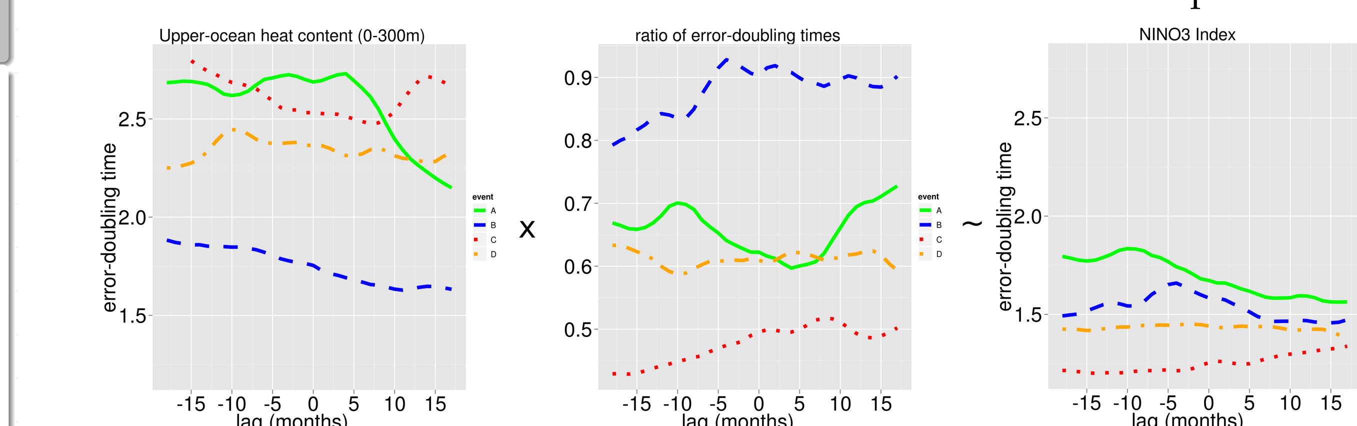
- The events are classified differently by the LLEs.

The ratio of the error-doubling times:

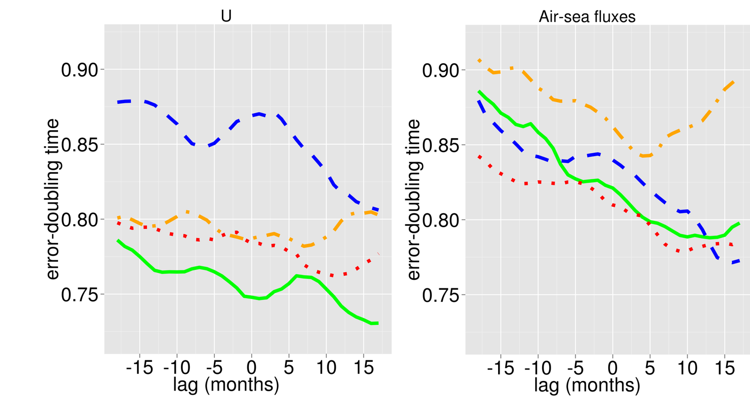
$$ratio = \frac{\tau_{NINO3}}{\tau_{t300}} \qquad (4)$$

is a measure of the propagation of information between the upper-ocean heat content and the SST.

It seems that there is a 'de-coupling' between the predictability of the upper-ocean heat content and that of the SST for the events that are deemed the less predictable.



- **Who is responsible for this 'de-coupling'?**
- The mapping of heat-content predictability ('slow' variable) onto NINO3 predictability ('faster' variable) could be 'distorted' by the (lack of) predictability in the winds ('even faster' variable).



8. Conclusions & Discussion

We used Local Lyapunov Exponents to characterize varying ENSO predictability, as constrained by internal nonlinearities, in GFDL's CM2.1 long pre-industrial run.

- Predictability varies (multi)decadally by as much as 9-18%.
- 'Active' ENSO periods are more predictable than 'inactive' ones.
- The relation predictability-magnitude varies by epoch of distinct ENSO behavior.
- The ERSST.v3 dataset and ZC model appear to lose information less rapidly than the unforced CM2.1 GCM. *Does this reveal a discrepancy between real-world and GCM predictability of ENSO? Or does it arise from the in-filling techniques and external forcings present in the historical reconstruction?*

We then investigated the sources of predictability in '1997-98-like' events.

- To the extent that the LLE-derived classification reflects the physical evolution of individual events, decreased predictability seems associated with a 'de-coupling' of predictability between the upper-ocean heat content and the SST anomaly, likely resulting from the role of the air-sea interactions.
- The rich variability in ENSO behavior and predictability in the pre-industrial simulation, motivates the application of such methods in GHG-forced simulations. *Could changes in the attractor in future climate conditions manifest in regime-like changes in predictability?*

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