1. Problem Statement

Decadal variability of ENSO is present in historical and paleo records, and has been simulated by a hierarchy of dynamical and statistical models. Predictability of ENSO varies with forecast lead-times, amplitude of interannual ENSO variability, and with decade. The limits of predictability depend on the mechanisms responsible for ENSO irregularity (chaos, noise), and equilibration at finite amplitude (stability of atmosphere-ocean interactions) (Sarkar & Cane, 2010). In an early report, Abarbanel et al. (1991) note that "the real issue of predictability is whether the atmosphere-ocean system constitutes a chaotic dynamical system at all time-scales", and point out that "extremely long (1000 years or more) runs of coupled atmosphere-ocean models are required to study the issue of whether the [climate] system exhibits chaotic behavior at all time scales of interest (...)."

Fortunately, such investigations are gradually becoming possible in long runs of coupled GCMs, such as the GFDL CM2.1 pre-industrial run (Wittenberg et al. 2006; Wittenberg 2009). In this work, we investigate the limits of predictability in the context of dynamical systems theory. We use the Local Lyapunov Exponents (LLEs) of the NINO3 time series as a measure of predictability of the ENSO system in long control runs of GFDL CM2.1 and the ZC model. We explore the (multi)decadal variability of predictability and its links to the magnitude and frequency of events, as well as the possible culprits for this variability.

2. Ergodic Theory of Dynamical Systems.

- The dynamics of a complex system can be captured by time-delay embedding of a single variable.
- Global Lyapunov Exponents measure the average rate of divergence of nearby trajectories in the phase-space, i.e. they characterize the average predictability of the attractor.

\[
\text{Prediction Error: } \quad E(t) = E(0) \exp(\lambda t),
\]

where \( \lambda \) is the largest global exponent.

Local Lyapunov Exponents \( \lambda(x,L) \) measure the growth or decay over \( L \) time steps of a perturbation made around a point \( x \) of the attractor:

\[
L \to , \quad \lambda(x,L) \to \lambda_{\text{global}}
\]

Local error-doubling time:

\[
(\lambda(x,L))^2
\]

LLEs are useful in characterizing predictability locally in the attractor of a weakly chaotic system that likely passes through phases of increased or decreased predictability (like ENSO).

3. ENSO Predictability in CM2.1-1860

- Colors indicate levels of predictability, as determined by the LLEs.
- Green → blue → red = more → less predictable (in 9% increments)

Colors indicate levels of predictability, as determined by the LLEs.

4. Decadal Variability in Predictability

- Periods with sinusoidal moderate events (M2) are deemed the least predictable.
- Periods with irregular moderate events (M3) are deemed more predictable in the epoch that 'mimics' the post-1960 observations (M6).
- Periods with sinusoidal lesser events (M1) are deemed the most predictable.
- Periods with irregular lesser events (M4) are deemed less predictable in the epoch that 'mimics' the post-1960 observations (M6).

5. Predictability-Magnitude Relations vary by Epoch.

Nature seems to be setting the upper bound of predictability for the GCM. The intermediate ZC model has error-doubling times closer to observed, but less variable.

6. Model vs. Observations: Who sets the Upper Bound of Predictability?

- The median error-doubling time in the GCM (1.6 months) is less than the observed (22 months).
- Less variability in predictability in the model vs. observations.
- Predictability reduces from 1901-1960 to 1961-2000 by 8.5% (in the model 4%).
- But, heavier tails indicate cases of increased predictability in the record.

7. ‘1997-98 events’ in CM2.1: What dictates their predictability?

- The events are classified differently by the LLEs.
- The ratio of the error-doubling times:

\[
\text{ratio} = \frac{\text{LLE}}{\text{NINO3 Index}}
\]

It seems that there is a 'de-coupling' between the predictability of the upper-ocean heat content and that of the SST for the events that are deemed the less predictable.

8. Conclusions & Discussion

We used Local Lyapunov Exponents to characterize varying ENSO predictability, as constrained by internal nonlinearities, in GFDL’s CM2.1 long pre-industrial run.

- Predictability varies (multi)decadally as much as 9-18%.
- “Active” ENSO periods are more predictable than ‘inactive’ ones.
- The relation predictability-magnitude varies by epoch of distinct ENSO behavior.

We then investigated the sources of predictability in 1997-98-like events.

- To the extent that the LLE-derived classification reflects the physical evolution of individual events, decreased predictability seems associated with a ‘de-coupling’ of predictability between the upper-ocean heat content and the SST anomaly, likely resulting from the role of air-sea interactions.
- The rich variability in ENSO behavior and predictability in the pre-industrial simulations motivates the application of such methods in GCM forced simulations.
- ‘Active’ periods (‘M3’ vs ‘M4’) are on average more predictable than ‘inactive’ ones (‘M1’ vs ‘M6’).

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